The Theory of Meccano Gears

Part 1: Spur gears

1) Definition: Spur gear

A gear having straight teeth formed on a cylindrical surface, parallel to the axis of rotation. Spur gears are used to transmit drive across parallel shafts. Figure 1 shows a typical Meccano spur pinion (1) and spur gear (2).

Fig. 1: Typical Meccano Spur Pinion (1) & Spur Gear (2)

2) Introduction

Gearing is sometimes regarded as a 'Black-art' or at least an abstruse subject. Like most technical subjects, it can become very complex, especially when dealing with multi-dimensional curved surfaces described by mathematical or geometric laws of curves, such as used in the design of spiral, hypoid and spiroid gears for instance. However, for the straight-cut Meccano spur gears, there should be no reason for not designing them to a coherent 'system' and producing them to normal standards.

In Meccano publications that I have read, there have been some suggestions that Meccano gears are not of involute form, that some gears are "peculiar", "pointed" or at least produced by experimental methods. Not having any inside knowledge of how Meccano gears were or are made, I prefer to assume that they were designed and manufactured using gear technology and well established engineering practice and principles. My examination and measurement of Meccano gears over the years supports the above assumptions, also indicating that they conform to a British Standard Specification (ref. 1), notwithstanding a few poor quality or improperly finished gears.

I would like to present and share my experience and understanding of Meccano gears in this article, starting with the basics in applying gear theory and practice to the Meccano range of spur gears. I have no knowledge of other Meccano look-a-like systems, so these are not considered. I use the noun "gear" as a generic term and also, as in Meccano terminology, for the larger of a pair of mating gears. Where necessary, to distinguish the difference, I use the term "pinion" for the smaller gear.

A gear is merely a continuously rotating lever in the form of a cylinder, with bumps, pegs, pins or teeth applied to the surface to prevent slipping. For maximum rolling efficiency, formed teeth have proved successful. Many geometric curves, or forms, have been considered and applied, but the involute form has become the most popular tooth profile, satisfying efficiency and ease of manufacture.

Gears are made using a multitude of methods such as forging, stamping, rolling and machine cut by shaping, hobbing, shaving and grinding etc. For the purpose of this article, I refer to machine cut methods, since these help explain the flexibility of the generated involute tooth form. Gear tooth features and terminology need to be considered in the explanation and understanding of this fascinating subject. Often standard values or ranges are adopted for reasons of standardization of tooling, interchangeability, ease of manufacture, convenience, proven good tooth meshing action or strength and wear durability. However, standard proportions are by no means strict rules, as, in particular industries, departure from these can produce optimum advantage in application.

There is no particular order of preference in selecting and defining gear geometric variables in the design process, this may depend on the application and specific requirements, ie: such as if the gear is to 'fit' within an existing dimensional system or constraint, if it is required to satisfy a particular ratio requirement, if it is required to operate at high speed or load, if it is required to meet low noise emission or satisfy high levels of safety or reliability etc, Any of these requirements may define some of the starting parameters of gear design, fortunately, not all are important or necessary in the design of Meccano gears.

3) Gear proportions and definitions

In the following nomenclature, capital letters refer to the gear and lower case to the pinion. Where functions can apply to both, I have referred only to the gear. Common abbreviations are shown in light parenthesis followed by any mathematical nomenclature applicable. The equations are numbered in parenthesis for further reference. Figure 2 shows the definitions used.

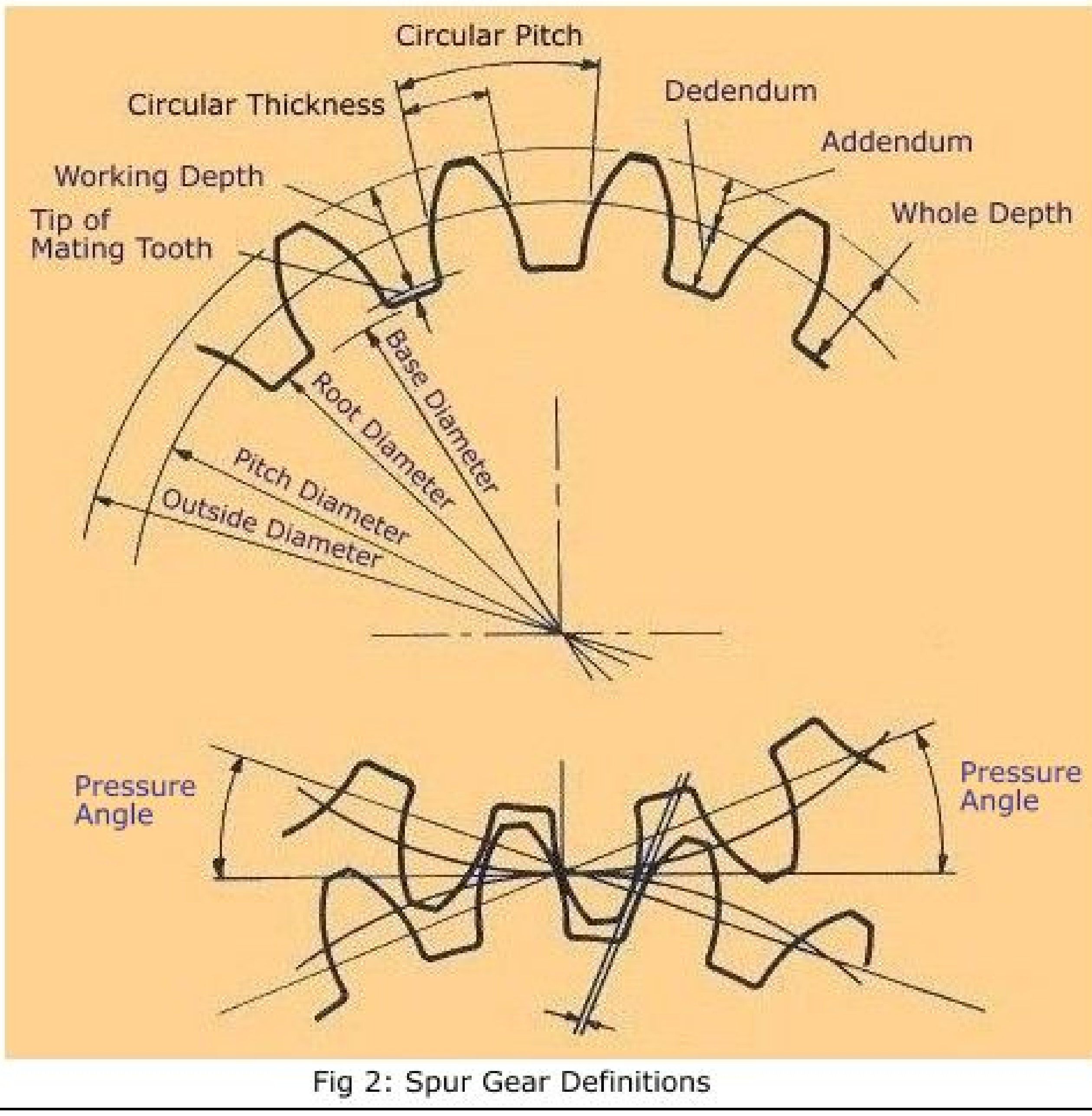


Fig 2: Spur Gear Definitions

The usual order of selecting and defining gear design variables is as follows (with some reasoning, although advanced reading may be necessary to identify all variables): -

Pitch circle (PCD) (**D**)

This is the diameter of a circle about which the gear tooth geometry is constructed. In gears having standard proportions, it is known as the standard pitch circle and represents the imaginary circles or cylinders of each gear in rolling tangential contact, their diameters being in direct proportion to the number of teeth in each gear respectively and hence the gear ratio.

$D = N/P$ (1)

Operating pitch circle (D_e)

Sometimes standard proportions cannot be adhered to; in this case the gear tooth may be 'shifted' from its relationship with the standard pitch circle. Then a new imaginary "operating pitch circle" is established, which has the same tangential contact with the operating pitch circles of its mating gear(s) and as before, the same relationship to tooth numbers and ratios.

$D_e = 2NC_e/(N+n)$ (2a) or: $d_e = 2nC_e/(N+n)$ (2b)

Ratio (**R**)

In designing gear drives, the speed or torque relationship between the driven and driving shafts is generally known, or at least target values are estimated. The required ratio is then expressed as the input speed divided by the output speed in relation to unity. This ratio is then used to determine the gear sizes in terms of pitch circle diameters using:

$R = D/d$ (3a) or: $D = Rd$ (3b) or: $d = D/R$ (3c)

Pitch

The pitch of gear teeth is a measure of their distance apart around the pitch circle. It is expressed in the following forms: -

Diametral Pitch (DP) (**P**) - the number of teeth per inch of diameter.

$P = N/D$ (4a) or: $D = N/P$ (1) or: $N = PD$ (4b)

Module (**m**) - (metric pitch) the number of millimetres diameter per tooth.

$m = D(mm)/N$ (5a) or: $D = mN$ (5b) or: $N = D/m$ (5c)

Circular pitch (CP) (**p**) - the distance measured around the pitch circle from one tooth to the next.

$p = \pi D/N$ (6a) or: $D = pN/\pi$ (6b) or: $N = \pi D/p$ (6c)

Converting pitch

Some useful formula for expressing one pitch system in terms of another are: -

$p = \pi/P = \pi m/25.4$ (7a) $P = \pi/p = 25.4/m$ (7b) $m = 25.4p/\pi = 25.4/P$ (7c)

For our purposes, we will use diametral pitch, as this is the known basis of proportioning Meccano spur gear teeth, ie: 38 DP (P = 38). However, Meccano spur gears are: 0.08267" circular pitch - from (7a), or 0.66842 metric pitch module - from (7c).

Number of teeth (N or n)

The number of teeth in the gear (N) or pinion (n).

Where: $N = DP$ (8a) &: $n = dP$ (8b)

If these are integers, then the gears will have standard proportions. As is more likely, they will have decimal (or fractional) values, and then standard values may be derived by adjusting the pitch diameters to the nearest integer tooth values. To do this, new pitch diameters are calculated using the nearest integer values to N & n above,

ie: $D = N/P$ (9a) &: $d = n/P$ (9b)

This will compromise the centre distance and the ratio. If this compromise is not acceptable, then more trial iterations are made in an attempt to satisfy the important parameters, otherwise non-standard proportions may be used (more about this later). Of course, the choice of values for N & n, may also be driven by the ratio requirements when:

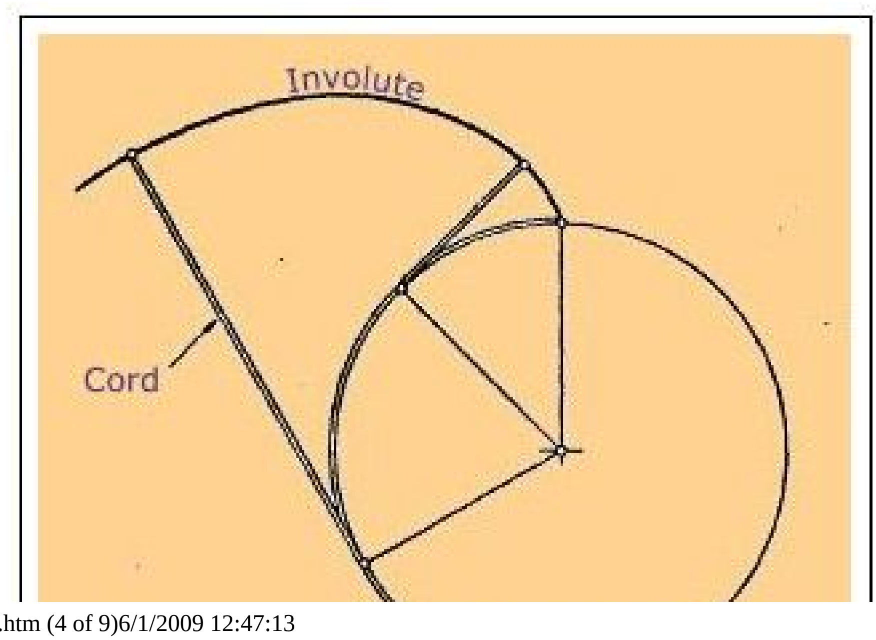
$R = N/n$ (10a) or: $N = Rn$ (10b) or: $n = N/R$ (10c)

Base circle (D_b)

The base circle is a circle of diameter equal to the cosine of the pressure angle times the pitch diameter,

ie: $D_b = D\cos\psi$ (11)

from which the involute curve forming the tooth profile is generated.



Involute

The involute curve is generated by the locus of a point on a piece of string or cord (for example), when un-wound from a cylinder equivalent in diameter to the base circle, see figure 3. A part of this involute curve forms the gear tooth profile between its junction with the root radius and the gear outside diameter.

Fig 3: Involute Curve Generation

Line or path of action

An imaginary line tangent to and joining the base circles of a pair of mating gears. It is the line along which the point of contact passes, as a pair of teeth enters and leaves mesh. It can also be considered as a cord, wrapped around both base circle cylinders, transmitting drive from one to the other. See figure 4.

Pitch point

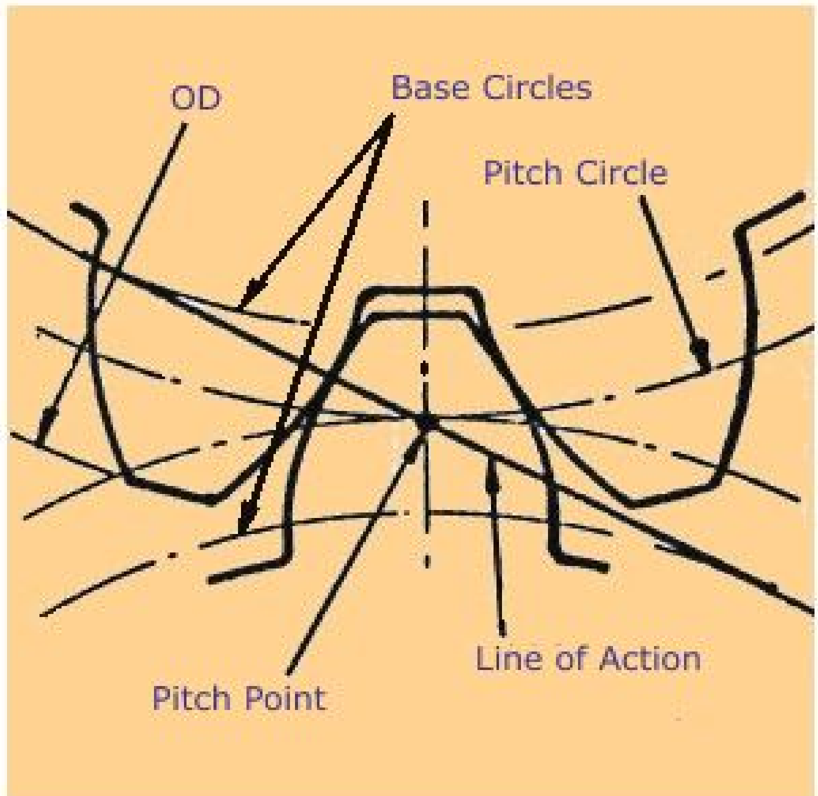


Fig 4: Pitch Point & Line of Action Geometry

The point at which the line of action intersects a line passing through the axes of both gears.

Pressure angle (PA) (ψ)

The pressure angle is the angular deflection of the line of action normal to the centreline through the gear axes. For standard cut gears it is also the pressure angle of generation (PA of cutter). It can be varied to alter the tooth contact conditions (contact ratio) or tooth bending strength.

A typical value is 20°, a standard value for which a tremendous amount of tooling exists in the gear manufacturing industry. A smaller angle increases contact ratio and decreases strength, a larger pressure angle has the converse effect. Early gears used 14-1/2° pressure angle, mainly because the sine of 14-1/2° is very nearly 1/4, which made calculations easier before the advent of electronic calculators. 25° PA is sometimes used for increased tooth strength at the expense of higher tooth separating loads. The pressure angle can be any practical value to satisfy specific requirements or special purpose gearing.

Some time ago I was able to examine a selection of Meccano gears using an engineering inspection machine called a shadowgraph. This machine produces sharp edge magnified images, projected onto a screen for comparison with templates. I made a template of a 57 tooth gear tooth profile 20 times full size and compared it with the projected image of an actual Meccano gear. The result confirmed to me that the Meccano gear was of involute form having a 20° pressure angle (also conforming with ref. 1).

Operating pressure angle (ψ_e)

The same as above for standard gears, but different for gears designed to work at extended or contracted centre distances. The operating pressure angle can be calculated from the following equation if standard PCD's, pressure angle of generation and extended centre distance are known: -

$\psi_e = \text{arc}\cos[(D+d)\cos\psi/2C_e]$ (12)

Centre distance - standard (C)

This is the distance between the operating centres of a pair of mating gears. It can be calculated from:

$C = (N+n)/2/P$ (13)

Extended centre distance (C_e)

When a pair of gears are required to operate at a pre-determined centre distance, gears designed to standard proportions will not always 'fit' that centre distance. In this case, it is usual to choose standard pitch and tooth number proportions (to satisfy, or approximate the required ratio), which fall close to, but less than, the target centre distance. Then, the involute profile of one or both gears is shifted, so as to operate at the "extended centre distance". This procedure is normal and very important in designing gears around standard tooling. Applying positive displacement to the involute profile (shifting it radially outwards) has the advantage of thickening the tooth at its root, providing additional strength.

Gears can also be designed to work at "contracted (or closed) centre distance", this applies when a pair of gears with particular tooth numbers are needed, but the calculated standard centre distance is greater than that required. In this case one or both gears are cut with negative profile shift, ie: the teeth are thinned to allow closer than standard meshing centres. Examples of Meccano gears, cut with standard, positive and negative profile shift, using the centre distance equation (13), are illustrated as follows: -

(19+57)/76 = 1"; (19+95)/76 = 1.5"; (19+133)/76 = 2"; (38+38)/76 = 2" - these are cut to standard tooth thickness (less backlash allowance of course).

(25+50)/76 = 0.9868"; (15+60)/76 = 0.9868"; (30+45)/76 = 0.9868" - these combinations require positive profile shift (centre distance extension) to mesh at 1.00" centres, although, the small difference in calculated centre distance of 0.013" (equivalent to approximately 4.4 thou' backlash) may equate to the amount of thinning for backlash required.

(22+55)/76 = 1.013"; (11+66)/76 = 1.013"; (13+65)/76 = 1.0263" - require negative profile shift (centre distance contraction) to mesh at 1.00" centres.

A commercial alternative is to optimize the pitch to a non-integer, non-standard value which may require special cutters or tooling for manufacture. This would be an entirely acceptable solution for moulded or stamped gears for instance, or for a pair of gears not part of a 'system' requiring interchangeability of gears of different sizes, or if producing special cutters for machine cutting were economically viable.

Circular Tooth thickness (t)

This is the tooth thickness measured around the pitch circle diameter (circular measure), between the points where the involute curves cross the pitch circle diameter. It follows therefore, that if the circular tooth thickness is the same as the circular tooth space (no backlash), then the sum of the tooth thicknesses and tooth spaces equal the pitch circle circumference, leading to the expression:

$t = \pi D/N/2 = \pi/2/P$ (14)

The tooth thickness is an important variable in modifying gear teeth to meet the requirements of non-standard operating conditions. Probably the most practical purpose for varying the tooth thickness is "thinning for backlash". In this instance, the gear tooth cutter is 'fed-in' more than is required to produce the standard thickness, which generates a thinner tooth to provide running clearance, or "backlash". The thinning for backlash can be applied to one or both gears of a mating pair. When applied to both, the allowance is shared between the two (not necessarily equally). Sometimes all the thinning for backlash is removed from one gear of a pair, usually the wheel. This is done for better balancing of tooth strength, because the larger gear (wheel) has wider teeth at the root and lower tooth mesh frequency than the pinion.

Addendum (a)

The radial height of a gear tooth above the standard pitch circle. Its value is:

$a = 1/P$ (15)

Outside diameter (OD) (D_o)

The diameter of a gear over the tips of its teeth. Sometimes referred to as the "major" diameter. The standard OD is:

$D_o = D + 2a$ (16)

This standard OD is usually the size of the gear blank, prior to cutting the teeth. The OD can be made non-standard within certain limits. A larger OD increases contact ratio, smaller (sometimes referred to as "topping" the teeth) reduces contact ratio, but may be desirable for providing clearance to adjacent parts.

Dedendum (d)

The radial depth of a gear tooth below the standard pitch circle. This is greater than the addendum to allow for clearance of the tips of the mating gear teeth to the root radius, or fillet form. It varies, in standard form, dependant upon the cutting method. My measurements of Meccano gears indicates their conformance with ref.1, which is:

d = 1.4/P (17)

When the cutter is displaced for a non standard gear, the dedendum varies accordingly.

Whole depth of tooth (**h**)

The radial height of a gear tooth between the root and outside diameters. Also controlled by cutter proportions and depth of cut. The standard whole depth is a+d or:

h = 2.4/P (18)

Root diameter (D_r)

The diameter measured between the tooth root fillet radii, sometimes referred to as the "minor" diameter. This is a function of the depth of cut produced by the cutter. The cutter will be proportioned to satisfy the dedendum requirements. The standard root diameter is:

D_r = D - 2.8/P (19a) or: D_r = Do - 4.8/P (19b)

Fillet radii

The radius or radii in the tooth root between the involute flanks and below the active profile. This is made as large as possible for increased tooth bending strength and minimum stress concentration.

Active profile

That part of the tooth involute flanks over which the point of contact passes.

Tooth flank

The contacting faces of gear teeth.

Backlash

The shortest distance (gap) between the involute flanks of a mating pair of gears on the non- contacting flanks (the running clearance).

Face width (FW)

The axial width of a gear at the toothed portion.

Contact ratio (CR)

The amount that sequential tooth meshes overlap. For smooth, quieter operation, more than one tooth must be in contact with teeth of the mating gear. Typical values are: 1.1 to 1.8. It can be increased by modifying the addendum (long addendum).

4) Gear Design

Reference 3 is a spur gear design programme, based on the analyses used in reference 2, and was used to confirm the theory and conclusions I reached in this study and measurement of Meccano gears.

References:

1) British Standard - BS 978 : Part 1 : 1952 - GEARS FOR INSTRUMENTS AND CLOCKWORK MECHANISMS.

2) "GEAR ENGINEERING - Dr H E MERRITT"

3) Computer spreadsheet 'spurgearcalc' by AWenbourne

Please direct any queries or comments to the "Other / General Resources / Prototype Information & Research" section of the Meccanoscene public forum.

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